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## **Extensive and intensive margins and the choice of exchange rate regimes**

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# Extensive and intensive margins and the choice of exchange rate regimes

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## Abstract

This paper studies how the choice of fixed or flexible exchange rate regimes is affected by the existence of intensive and extensive margins. We study two models where firms enter during or before each period of production. We show how the choice of those regimes depend on the level and the volatility of the intensive and extensive margins as well as on the congruence between consumers' preferences and the supply and diversity of products. We show that fixed exchange rate regimes are preferred for high enough labor supply elasticities. Fixed exchange rate regimes are unambiguously better when entry occurs at the same time as production in each period. Fixed exchange rate regimes are less attractive in the presence of production lags and higher love of product diversity.

**Keywords:** firm entry, product diversity, exchange rate system

**JEL classification:** E22, E52, L16

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# 1 Introduction

Since Friedman (1953), the international and monetary economics literature has widely studied the transmission of international shocks within the production sector. Accordingly, the adoption of a pegging policy or a common currency union shifts most adjustments to the real side of the economy. Flexible exchange rates, on the other hand, correct relative price misalignments and absorb macroeconomic shocks. Adjustments on the real side of the economy consist not only of the expansion and contraction of existing firms' production - intensive margins -, but also the entry and exit of firms - extensive margins. The choice of fixed or flexible exchange rate regimes is therefore likely to be affected by the way those margins respond to shocks. While previous literature has mainly focused on the role of intensive margins on this choice, the present paper aims to study the role of extensive margins and to compare them with the literature.

This paper discusses the costs and benefits of fixed and flexible exchange rate regimes in a general equilibrium model that includes two countries, money holdings, elastic labor supply, stochastic exogenous demand shocks and endogenous intensive and extensive margins. Firms produce and sell differentiated products under monopolistic competition. They enter or exit by comparing operational profits with costs of entry. Wages are assumed to be sticky for one period. Under such nominal rigidities, money has a role in the economy beyond a mere unit of account. Thus, monetary policy may have an impact on both extensive as well as intensive margins. Without any state-contingent financial assets internationally held, the choice of exchange rate regimes becomes critical for welfare.

Accepting closed form solutions, the model allows us to discuss the effect of asymmetrical demand shocks on intensive and extensive margins and on the choice of exchange rate regimes. We then study two repeated entry models where firms enter at every time period. In the first model, firms enter and incur a fixed cost investment in the same period as they produce (contemporaneous production). In the second model, firms must enter and invest one period in advance to their production period (lagged production). In this case, consumer may find an alternative to money saving by lending their funds to firms while there may exist a discrepancy between shock realizations and product supply realizations.

We can preview our results as follows. In general, the loss of the exchange rate instrument requires adjustments in extensive as well as intensive margins following demand shocks. First, when entry is contemporaneous to production, adjustments arise only in extensive margins and in the fixed exchange rate system. Extensive margins are procyclical and imply lower levels and higher volatility in extensive margins. Yet, such procyclical movements in extensive margins

raise welfare because they occur at the very moment of the demand shifts. The choice of a fixed exchange rate regime must therefore account for its costs (the lower mean level of and higher variability in extensive margins) and its benefits (the stronger congruence between preference and product diversity). These costs decrease with higher labor supply elasticities. Second, when firms enter and invest before they produce, both margins fluctuate under fixed exchange rates. Intensive margins fluctuate according to the current demand shocks while extensive margins vary with one period of lag. The changes in intensive margins leads to consumption levels that are congruent with demands, which benefits consumers. However, the volatility in extensive margins reduces consumers' welfare. As in the contemporaneous entry model, we show that fixed exchange rate regimes are preferable for high enough labor supply elasticities. More importantly, we also show that fixed exchange rate regimes are supported for a *smaller* set of parameters when production is lagged behind investment. This is further the case when consumers express a higher preference for product diversity. The extensive margin volatility has indeed a higher impact on households' welfare when they attach a higher importance to product diversity. In those two models, extensive margins always operate as a shock absorber that substitute for exchange rates (in a perfect or imperfect way respectively in the first or second model).

In this paper, there is neither international borrowing and lending nor fiscal transfer.<sup>1</sup> Risk sharing across countries is therefore imperfect and the flexible price allocation realized under flexible exchange rate regimes deviates from that obtained under complete asset markets.<sup>2</sup> The Pareto efficient allocation would have the product supply and diversity set according to changes in the tastes for each country's products and would balance those margins efficiently across countries. However, although flexible exchange rates correct relative prices and realize higher average production and lower volatility, they fail to ensure that product supply and diversity align with consumer demands. By contrast, fixed exchange rate regimes have the opposite properties.

Our contribution relates to the literature in the following way. First of all, our model and results can be compared with Devereux's (2004) contribution qualifying the prevailing view that

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<sup>1</sup>This corresponds to situations where governments are unable to organize significant transfers between countries and where households and/or governments are unable to use the international credit market in the long run because of various borrowing constraints (e.g. credit constraints for households, Maastricht treaty for E.U. countries, I.M.F. constraints for developing countries). The 2011 Greece-EU crisis is a good illustration of the difficulties of organizing international transfers and access to credit markets.

<sup>2</sup>The deviation is called a "demand imbalance" in the literature (Corsetti et al. 2010a, 2010b). See also Hamano (2009a, 2009b) for related topics of risk sharing and extensive margins.

exchange rates are the most important shock absorbers. Discussing a static economy with two countries, two varieties, wage rigidities and constant returns to scale, Devereux shows that fixed exchange rates improve welfare compared to flexible ones when the elasticity of labor supply is sufficiently high. Our model extends this model to full dynamics, increasing returns to scale, monopolistic competition and the entry and exit of firms. The extension allows us to discuss the role of the balance between intensive and extensive margins in welfare ranking. Comparing results, it turns out that, when production and investment occur in the same time period, the choice of an exchange rate regime is the same in Devereux (2004) as our model. The forces at play are, however, different since product substitutability and love of product diversity drive the welfare effects of extensive margins in this paper. Our results differ when firms invest in advance to production. This paper also presents and discusses a formal decomposition of the welfare contribution of the two margins.

The present paper emphasizes the mechanisms through which intensive and extensive margins can be accountable for the choice of exchange rate regimes. Our approach contrasts with the classical literature, where firms' entry is driven solely by "real factors" such as productivity and population shocks (Krugman 1980, 1991; Melitz 2003; Ghironi and Melitz 2005). In this instance, Naknoi (2008a, 2008b)'s contribution is very close to ours. Naknoi analyzes how exchange rate regimes impact extensive margins through endogenous tradability based on a Ricardian comparative advantage.<sup>3</sup> While relocation of firms between tradable and non-tradable sectors arises in her model, our model accommodates free entry conditions and exporting by all firms. Because the sectorial relocation of firms is instantaneous in Naknoi's model, there exists no welfare cost that could arise from the mismatch between taste and product diversity under fixed exchange rate regimes. Baldwin and Nino (2006) and Bergin and Lin (2010) also look at the impact of a common currency on extensive margins. They, however, attach a special role to fixed costs and abstract away from monetary issues. Finally, our paper builds upon the so-called New Open Economy Macroeconomics (see, for instance, Obstfeld and Rogoff 1995, Corsetti et al. 2010b). While this literature has focused on optimal monetary policies under complete financial markets, it is beginning to investigate those policies under incomplete financial markets, as we do in this paper. This approach is also followed by Ching (2003) and Picard and Worrall (2009), who consider monetary transfers within currency unions that correct for the incompleteness of financial markets.

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<sup>3</sup>Interaction between extensive margins and monetary policy has been investigated in a closed economy. See for instance, Bergin and Corsetti (2008), Bilbiie, Ghironi and Melitz (2007), Lewis (2009) and Bilbiie, Fujiwara and Ghironi (2011).

The structure of the paper is as follows. Sections 2 and 3 present the model and discuss the equilibrium. Section 4 analyzes the case where firms enter, invest and produce in the same time period. Section 5 studies the case with production lags. Section 6 concludes.

## 2 Model

The present model discusses the welfare costs and benefits of exchange rate regimes between two countries, Home and Foreign. (Foreign variables are denoted with asterisks.) Each country is inhabited by a unit mass of households who are differentiated only in terms of their labor services. Wages are set by households one period in advance of production. There are no fiscal transfers and no borrowing and lending across countries. We describe the domestic country (Home). The same description holds for Foreign.

**Households** In every time period  $t$ , each household  $i \in [0, 1]$  consumes goods in a domestic set  $\mathcal{X}_t$  and a foreign set  $\mathcal{Z}_t$  of differentiated varieties. It also holds a quantity of money  $M_t(i)$  and supplies  $l_t(i)$  labor units (worked hours). The household maximizes its expected intertemporal utility,  $E_0 \sum_{t=0}^{\infty} \beta^t U_t$  where  $\beta \in (0, 1)$  is a common discount rate and where utility in period  $t$  is given by the following two-tier utility function:

$$U_t(i) = \ln C_t(i) + \chi \ln \frac{M_t(i)}{P_t} - \kappa \frac{[l_t(i)]^{1+\psi}}{1+\psi}$$

where

$$C_t(i) = \left( \frac{X_t(i)}{\alpha_t} \right)^{\alpha_t} \left( \frac{Z_t(i)}{1 - \alpha_t} \right)^{\alpha_t^*}$$

and

$$X_t(i) = N_t^{\gamma - \frac{1}{\sigma-1}} \left( \int_{\omega \in \mathcal{X}_t} x_t(i, \omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad Z_t(i) = N_t^{*\gamma - \frac{1}{\sigma-1}} \left( \int_{v \in \mathcal{Z}_t} z_t(i, v)^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}}$$

In this definition,  $P_t$  is the consumer price index, and  $M_t(i)/P_t$  is household  $i$ 's real money holding. The parameter  $\psi$  measures the inverse of the (Frisch) elasticity of labor supply while the parameters  $\chi$  and  $\kappa$  measure the intensity of preferences towards real money holdings and individual labor supply (worked hours) respectively. We call  $C_t(i)$  the composite bundle and  $X_t(i)$  and  $Z_t(i)$  the consumption baskets of domestic and foreign product. While  $x_t(i, \omega)$  denotes its consumption of domestic varieties  $\omega \in \mathcal{X}_t$ ,  $z_t(i, v)$  denotes the consumption of foreign varieties  $v \in \mathcal{Z}_t$ .  $N_t$  and  $N_t^*$  denotes the mass of domestic and foreign varieties. Under the above preferences, the parameter  $\sigma > 1$  measures the elasticity of substitution among varieties within the same consumption basket while  $\gamma \geq 0$  measures the preference (love) for product

diversity within each consumption basket (Benassy 1996).<sup>4</sup> The breakdown between product substitution and love for variety will be important in the discussion of the welfare impact of intensive and extensive margins. Finally, the parameters  $(\alpha_t, \alpha_t^*)$  where  $\alpha_t^* \equiv 1 - \alpha_t \in (0, 1)$  measures the preference between the two consumption baskets. Following Devereux (2004), we will consider that the economy is hit by demand shocks so that  $\alpha_t$  follows an i.i.d. stochastic process which is symmetrically distributed around a mean equal to  $1/2$ .

A Home household supplies its labor and earns an income  $w_t(i)l_t(i)$  where  $w_t(i)$  is its hourly wage. In addition to spending its income on the above product varieties and holding money, the household can invest in domestic firms. We consider two models. In the first, firm investment and entry occur in the same time period so that  $m = 0$ . In the second, investment occurs one period before entry as a result of share holding choice by households -that is  $m = 1$ . The household can therefore invest in a firm that produces the variety  $\omega \in \mathcal{X}_{t+m}$  that become available either within the same period ( $m = 0$ ) or within the next period ( $m = 1$ ). The household budget constraint is then given by

$$\begin{aligned} \int_{\omega \in \mathcal{X}_t} p_t(\omega) x_t(i, \omega) d\omega + \int_{v \in \mathcal{Z}_t} p_t(v) z_t(i, v) dv + \int_{\omega \in \mathcal{X}_{t+m}} s_t(i, \omega) q_t(\omega) d\omega + M_t(i) \\ = w_t(i)l_t(i) + \int_{\omega \in \mathcal{X}_t} s_{t-m}(i, \omega) d_t(\omega) d\omega + M_{t-1}(i) \end{aligned}$$

where  $p_t(\omega)$  and  $p_t(v)$  are the (domestic) prices of Home and Foreign varieties  $\omega \in \mathcal{X}_t$  and  $v \in \mathcal{Z}_t$ . In this expression  $q_t(\omega)$  denotes the price at date  $t$  for a share of a firm that enters at date  $t$  and produces variety  $\omega \in \mathcal{X}_{t+m}$  at date  $t + m$ , while  $d_t(\omega)$  denotes the dividend paid by an incumbent producer  $\omega \in \mathcal{X}_t$  at period  $t$ . Household  $i$  spends a share  $s_t(i, \omega)$  on the stock of an entering firm  $\omega \in \mathcal{X}_{t+m}$  and receives the share  $s_{t-m}(i, \omega)$  of the dividend paid by every incumbent producer  $\omega \in \mathcal{X}_t$ .

**Firms** Firms' activities are described as follows. Consider a Home firm that produces a differentiated variety  $\omega \in \mathcal{X}_t$  under increasing returns to scale and sells its products under monopolistic competition at date  $t$ . We assume that to produce its output the firm must spend on "establishment" activities (e.g. building a production plant) at the time period  $t - m$  where  $m = 0, 1$ . Every firm employs a set of horizontally differentiated labor services. To make this more precise, we assume that each household  $i \in [0, 1]$  offers a differentiated labor service and that every firm  $\omega$  demands the quantities of labor services  $\ell_t(i, \omega)$  and  $e_t(i, \omega)$ , for its

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<sup>4</sup>The parameter  $\gamma$  is equal to zero when consumers express no love for variety and to  $1/(\sigma - 1)$  when they have the preference for product variety that is assumed in Dixit and Stiglitz (1977).

production and setup activities, respectively. To produce  $y_t(\omega)$  units of outputs, the firm uses the set of labor services given by

$$y_t(\omega) = \left( \int_0^1 \ell_t(i, \omega)^{1-\frac{1}{\theta}} di \right)^{\frac{1}{1-\frac{1}{\theta}}} \quad (1)$$

whereas it uses

$$f = \left( \int_0^1 e_{t-m}(i, \omega)^{1-\frac{1}{\theta}} di \right)^{\frac{1}{1-\frac{1}{\theta}}} \quad (2)$$

for its establishment activities at date  $t-m$ . In those expressions,  $\theta > 1$  is the elasticity of substitution among the different labor services. The firm pays a dividend to its shareholders. This dividend is equal to the contemporaneous operational profit that includes sales and production cost:

$$d_t(\omega) = p_t(\omega) y_t(\omega) - \int_0^1 \ell_t(i, \omega) w_t(i) di \quad (3)$$

The setup cost is equal to  $\int_0^1 e_{t-m}(i, \omega) w_{t-m}(i) di$ .

**Markets and governments** When product markets clear, each firm's supply equals the demand for its variety by both domestic and foreign consumers:

$$y_t(\omega) = \int_0^1 x_t(\omega, i) di + \int_0^1 x_t^*(\omega, j) dj, \quad \omega \in \mathcal{X}_t$$

where the superscript  $*$  denotes foreign consumption and  $j$  denotes each foreign household. Similarly, when labor markets clear, each household's labor supply equals the demand by firms:

$$l_t(i) = \int_{\omega \in \mathcal{X}_t} \ell_t(i, \omega) d\omega + \int_{\omega \in \mathcal{X}_{t+m}} e_t(i, \omega) d\omega, \quad i \in [0, 1]$$

In equilibrium, trade must be balanced so that the value of domestic imports equates the value of exports. We get

$$\int_0^1 \int_{\nu \in \mathcal{Z}_t} \varepsilon_t p^*(\nu) z_t(\nu, i) d\nu di = \int_0^1 \int_{\omega \in \mathcal{X}_t} p(\omega) x_t^*(\omega, j) d\omega dj$$

where  $\varepsilon_t$  is the exchange rate (namely, the price of one unit of foreign money in terms of the domestic currency),  $p^*(\nu)$  is the price of foreign variety  $\nu \in \mathcal{Z}_t$  denominated in the foreign currency, and  $x_t^*(\omega, j)$  is the foreign demand for the domestic variety  $\omega \in \mathcal{X}_t$ .

Finally, the central bank supplies an amount of money  $M_t$ . When the money market clears, the money supply is equal to its demand so that

$$M_t = \int_0^1 M_t(i) di$$



Symmetric conditions hold for the foreign country.

Wages are sticky during one time period. We define the equilibrium as follows: (i) each household  $i$  chooses a plan of money holding  $\{M_t(i)\}_{t=0}^\infty$ , consumption profiles  $\{x_t(i, \cdot), z_t(i, \cdot)\}_{t=0}^\infty$ , stock market positions  $\{s(i, \cdot)\}_{t=0}^\infty$  and wages  $\{w_{t+1}(i)\}_{t=0}^\infty$  applying in the next period, that maximize its intertemporal utility subject to its per-period budget constraint, (ii) each firm  $\omega \in \mathcal{X}_t$  chooses its product price  $p_t(\omega)$  and its labor demands  $\ell_t(i, \omega)$  and  $e_t(i, \omega)$  that maximizes its profit, (iii) the local stock market clears so that firms enter as long as they raise a stock price  $q_t(\omega)$  that meets future expected dividends and (iv) products, labor and money markets clear in every period. The money supply is set by each central bank with the objective of either a fixed or flexible exchange rate.

### 3 Equilibrium

We here describe the equilibrium choices by households and firms and determine the market equilibrium conditions for any exogenous monetary policy. Equilibrium conditions will be applied to the monetary policies of fixed and flexible exchange rate regimes in the next sections. For the sake of conciseness, we here discuss the cases of contemporaneous and lagged production together, equilibrium conditions being identical or similar. We finish by discussing equilibrium welfare.

**Household choices** In period  $t$ , the household  $i$  chooses its consumption profiles  $(x_t(i, \cdot), z_t(i, \cdot))$ , money holding  $M_t(i)$  and share holdings  $s_t(i)$ . First, its optimal consumption of home and foreign varieties can be computed as

$$x_t(i, \omega) = N_t^{\gamma(\sigma-1)-1} \left( \frac{p_t(\omega)}{P_{X,t}} \right)^{-\sigma} X_t(i) \quad \text{and} \quad z_t(i, \nu) = N_t^{*\gamma(\sigma-1)-1} \left( \frac{p_t(\nu)}{P_{Z,t}} \right)^{-\sigma} Z_t(i)$$

where

$$X_t(i) = \alpha_t \frac{P_t C_t(i)}{P_{X,t}} \quad \text{and} \quad Z_t(i) = (1 - \alpha_t) \frac{P_t C_t(i)}{P_{Z,t}}$$

are the chosen consumption baskets and

$$P_{X,t} = N_t^{\frac{1}{\sigma-1}-\gamma} \left( \int_{\omega \in \mathcal{X}_t} p_t(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{Z,t} = N_t^{*\frac{1}{\sigma-1}-\gamma} \left( \int_{\nu \in \mathcal{Z}_t} p_t(\nu)^{1-\sigma} d\nu \right)^{\frac{1}{1-\sigma}}$$

are the price indexes for those baskets. Finally, the consumer price index is given by

$$P_t = P_{X,t}^{\alpha_t} P_{Z,t}^{1-\alpha_t}$$

Second, the household's optimal money holdings and share of stock are expressed as the following real money demand equation.

$$\frac{M_t(i)}{P_t} = C_t(i) \frac{\chi}{1 - E_t \Lambda_{t,t+1}(i)} \quad (4)$$

for  $m = 0$  and 1, where

$$\Lambda_{t,t+1}(i) = \beta \frac{P_t C_t(i)}{P_{t+1} C_{t+1}(i)} \quad (5)$$

denotes the endogenous discount rate between  $t$  and  $t + 1$ .

While the equilibrium share of stock should be such that stock prices equal dividends ( $q_t(\omega) = d_t(\omega)$ ) when  $m = 0$ , the optimal share of stock is given by the following Euler equation when  $m = 1$ :

$$q_t(\omega) = E_t \Lambda_{t,t+1}(i) d_{t+1}(\omega). \quad (6)$$

**Firms' decisions** Firms produce under monopolistic competition. Consider a firm  $\omega \in \mathcal{X}_t$  that chooses its price and labor demand at date  $t$ . It maximizes its dividend payment (3) with respect to  $p_t(\omega)$  and  $\ell_t(\cdot, \omega)$  subject to the production function (1). The cost-minimizing demand for labor services is then equal to  $\ell_t(i, \omega) = (w_t(i)/W_t)^{-\theta} y_t(\omega) \forall i$  where

$$W_t \equiv \left( \int_0^1 w_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

is the wage index, common for all domestic firms. Considering the following iso-elastic demand for its variety

$$y_t(\omega) = \int_0^1 x_t(i, \omega) di + \int_0^1 x_t^*(j, \omega) dj = N_t^{\gamma(\sigma-1)-1} \left( \frac{p_t(\omega)}{P_{X,t}} \right)^{-\sigma} \left[ \int_0^1 X_t(i) di + \int_0^1 X_t^*(j) dj \right] \quad (7)$$

the firm sets its optimal price

$$p_t(\omega) = \frac{\sigma}{\sigma - 1} W_t \quad (8)$$

which is the same for all domestic varieties. Taking into account the above conditions, the firm's dividend is equal to  $d_t = p_t(\omega) y_t(\omega) / \sigma$ .

Firm  $i$  also minimizes its setup cost  $\int_0^1 e_{t-m}(i, \omega) w_{t-m}(i) di$ . Its cost-minimizing demand for labor services is equal to  $e_{t-m}(i, \omega) = (w_{t-m}(i)/W_{t-m})^{-\theta} f$ , which results in a cost  $W_{t-m} f$ .

**Wage setting** In this paper we consider wages that are sticky. The households set their wages one time period in advance. Accordingly, at date  $t$ , a domestic household  $i$  sets the wage  $w_{t+1}(i)$  that maximizes its expected utility  $E_t U_{t+1}(i)$  subject to next period's budget constraint and next period's balance between labor supply and demand:  $l_{t+1}(i) =$

$\int_{\omega \in \mathcal{X}_{t+1}} \ell_{t+1}(i, \omega) d\omega + \int_{\omega \in \mathcal{X}_{t+1+m}} e_{t+1}(i, \omega) d\omega$ . In the latter expression, the first and second terms respectively represent the labor demands for production activities at date  $t+1$  and for setup activities at date  $t+1$  by the firms producing at date  $t+1+m$ ,  $m = 0, 1$ . As seen above, this labor demand is a iso-elastic function of  $w_t(i)$ . One can show that the household sets its wage such that its expected disutility of a marginal work effort,  $\kappa \theta w_{t+1}(i)^{-1} E_t l_{t+1}(i)^{1+\psi}$ , equals its expected utility from the associated increase in consumption,  $(\theta - 1) E_t [l_{t+1}(i) / (P_{t+1} C_{t+1}(i))]$ . Hence,

$$w_{t+1}(i) = \kappa \frac{\theta}{\theta - 1} \frac{E_t l_{t+1}(i)^{1+\psi}}{E_t [l_{t+1}(i) / (P_{t+1} C_{t+1}(i))]} \quad (9)$$

**Local stock market equilibrium** The equilibrium in the local stock market depends on investment timing. When a firm enters and establishes production in the same period as its sales ( $m = 0$ ), it asks a stock price of  $q_t(\omega) = W_t f$  and pays a dividend of  $d_t(\omega) = p_t(\omega) y_t(\omega) / \sigma$ . Since  $q_t(\omega) = d_t(\omega)$  when  $m = 0$ , the stock market clears when

$$\frac{p_t(\omega) y_t(\omega)}{\sigma} = W_t f \quad (10)$$

By contrast, when a firm enters and invests in the period before its sales ( $m = 1$ ), it asks a stock price of  $q_{t-1}(\omega) = W_{t-1} f$  and pays a dividend of  $d_t(\omega) = p_t(\omega) y_t(\omega) / \sigma$ . The Euler equation (6) becomes

$$W_{t-1} f = E_{t-1} \Lambda_{t-1,t} p_t(\omega) y_t(\omega) / \sigma \quad (11)$$

**Product market equilibrium** The above analysis shows that households make the same choices and firms the same decisions within each country. This symmetry allows us to dispense with household- and firm-specific notations. We can now drop the reference to  $i$  and  $(\omega, \nu)$  without any ambiguity. The product market clear when trade is balanced. The balanced trade condition yields the exchange rate,

$$\varepsilon_t = \frac{(1 - \alpha_t) P_t C_t}{\alpha_t P_t^* C_t^*}, \quad (12)$$

which simply compares the value of imports (numerator) to the value of exports (denominator). Given the above relationships, we can readily determine the dividends and wages as well as the extensive and intensive margins.

Let  $N_t$  be the mass of firms  $\omega \in \mathcal{X}_t$  that produce in the domestic country. With the above balanced trade condition, each domestic firm's dividend is successively given by

$$d_t = \frac{1}{\sigma} p_t y_t = \frac{1}{\sigma} \frac{\alpha_t}{N_t} [P_t C_t + \varepsilon_t P_t^* C_t^*] = \frac{1}{\sigma} \frac{P_t C_t}{N_t}. \quad (13)$$

**Wages, extensive and intensive margins** On the one hand, consider a firm that enters and sets its production up in the same period as its sales ( $m = 0$ ). We know that  $d_t = p_t y_t / \sigma = W_t f$  so that the extensive and intensive margins are given by

$$N_t = \frac{1}{\sigma} \frac{P_t C_t}{W_t f} \quad \text{and} \quad y_t = \frac{\sigma W_t f}{p_t} = (\sigma - 1) f \quad (14)$$

As usual in Dixit-Stiglitz models, the intensive margin is constant while the extensive margin absorbs all shock variability. The equilibrium labor supply is determined as follows. Consider the wage and labor services supplied in period  $t + 1$ . The labor market clears if

$$l_{t+1} = N_{t+1} \ell_{t+1} + N_{t+1} e_{t+1} = N_{t+1} y_{t+1} + N_{t+1} f = \frac{P_{t+1} C_{t+1}}{W_{t+1}}$$

where we successively used (1), (2) and (14). Plugging this value of labor supply into the wage equation (9) yields the wage index

$$W_{t+1} = \left( \kappa \frac{\theta}{\theta - 1} \right)^{\frac{1}{1+\psi}} \left[ E_t (P_{t+1} C_{t+1})^{1+\psi} \right]^{\frac{1}{1+\psi}}$$

*Ceteris paribus*, the wage increases if the disutility from work increases (higher  $\kappa$ ) or their labor services become weaker substitutes (lower  $\theta$ ). In addition, it increases with a higher volatility of future nominal expenditures  $P_{t+1} C_{t+1}$  when  $\psi > 0$ .

On the other hand, consider a firm that enters and invests in the period before its sales ( $m = 1$ ). Then, the dividend is given by  $d_t = p_t y_t / \sigma$  and the share price by the Euler equation (11). The latter condition yields the extensive margin as

$$N_t = \frac{E_{t-1} \Lambda_{t-1,t} P_t C_t}{W_{t-1} f} = \frac{\beta}{\sigma} \frac{P_{t-1} C_{t-1}}{W_{t-1} f}, \quad (15)$$

while the former yields the intensive margin

$$y_t = \frac{\sigma - 1}{\sigma} \frac{P_t C_t}{W_t} \frac{1}{N_t}. \quad (16)$$

Both margins respond to shocks. In particular the extensive margin responds to the previous period's economic condition whereas the intensive margin responds to the current conditions. Note that the number of firms which appear as a consequence of households' consumption smoothing across time falls with more impatient investors (smaller  $\beta$ ).

When the labor market clears, the labor supply is equal to labor demand. So, using (1), (2), (15) and (16), we get the equilibrium labor supply as

$$l_{t+1} = N_{t+1} \ell_{t+1} + N_{t+2} e_{t+1} = N_{t+1} y_{t+1} + N_{t+2} f = \frac{\sigma - 1 + \beta}{\sigma} \frac{P_{t+1} C_{t+1}}{W_{t+1}}$$

Plugging this value of labor supply into the wage equation (9) yields the wage

$$W_{t+1} = \left( \kappa \frac{\theta}{\theta - 1} \right)^{\frac{1}{1+\psi}} \left( \frac{\sigma - 1 + \beta}{\sigma} \right)^{\frac{\psi}{1+\psi}} \left[ E_t (P_{t+1} C_{t+1})^{1+\psi} \right]^{\frac{1}{1+\psi}} \quad (17)$$

As above, the wage increases with higher disutility from work (higher  $\kappa$ ) and less substitutable labor services (lower  $\theta$ ). In addition, the wage increases with higher impatience (lower  $\beta$ ). This is because the extensive margin decreases, leading to a decrease in the remuneration from setup activities. At the limit, where  $\beta \rightarrow 1$ , the wage coincides to that under contemporaneous production assumption ( $m = 0$ ).

**Welfare** The household's utility is the sum of its utility from consumption and disutility from working

$$U_t^R = \ln C_t - \kappa \frac{l_t^{1+\psi}}{1+\psi},$$

and the utility from real money balance is

$$U_t^M = \chi \ln \frac{M_t}{P_t}.$$

Following Obstfeld and Rogoff (1995), we assume that the latter utility can be neglected ( $\chi \rightarrow 0$ ). The household's consumption can be computed as

$$C_t = \alpha_t N_t^{(1+\gamma)\alpha_t} N_t^{*(1+\gamma)\alpha_t^*} \left( \frac{y_t}{\alpha_t} \right)^{\alpha_t} \left( \frac{y_t^*}{\alpha_t^*} \right)^{\alpha_t^*}, \quad (18)$$

where  $\alpha_t^* = 1 - \alpha_t$ . This consumption level has the same expression in the contemporaneous production model. The role of preferences for product diversity is apparent. In the absence of love for diversity ( $\gamma = 0$ ), consumption increases with the total consumptions of domestic and foreign varieties,  $N_t y_t$  and  $N_t^* y_t^*$ . An increase in product diversity,  $N_t$  or  $N_t^*$ , does not impact household as long as it is compensated by a proportional decrease in consumption of each variety,  $y_t$  or  $y_t^*$ . However, in the presence of love for product variety ( $\gamma > 0$ ), consumption increases with  $N_t^{1+\gamma} y_t$  or  $N_t^{*(1+\gamma)} y_t^*$  so that an increase in product diversity increases welfare even if it is compensated by the same proportional decrease in consumption.

Note that, using the equilibrium labor supply and taking the expectation of  $U_t^R$ , the expected work disutility (second term in  $U_t^R$ ) is found to be constant and identical across exchange rate regimes. What matters for welfare is the expected utility from consumption (the first term in  $U_t^R$ ).

The intertemporal expected utility from consumption at period  $t = 0$  is equal to  $\sum_{t=0}^{\infty} \beta^t E_0 \ln C_t$  where

$$E_0 \ln C_t = E_0 \alpha_t \ln y_t + E_0 \alpha_t^* \ln y_t^* + (1 + \gamma) [E_0 \alpha_t \ln N_t + E_0 \alpha_t^* \ln N_t^*] + \text{cst}$$

Using symmetry, one can simplify the latter expression to  $2E_0\alpha_t \ln y_t + 2(1 + \gamma) [E_0\alpha_t \ln N_t] + \text{cst.}$  Developing those terms around expected output and product diversity, one further gets that the intertemporal expected utility from consumption depends on the following expression:

$$E_0\alpha_t E_0 \ln y_t + \text{cov}(\alpha_t, \ln y_t) + (1 + \gamma) [E_0\alpha_t E_0 \ln N_t + \text{cov}(\alpha_t, \ln N_t)] \quad (19)$$

This expression presents the welfare impact of variations in production and product diversity. The first and second terms reflect welfare effect of changes in consumption of each variety while the third term show the effect of changes in product diversity. For instance, fluctuations in intensive margins  $y_t$  impact expected utility through their mean and variance (first term) and through their covariance with demand shocks (second term). On the one hand,  $E_0 \ln y_t$  increases with output mean and falls with output variance.<sup>5</sup> On the other hand, the covariance increases with the congruence between consumers' preferences and the quantity of each product variety supplied to the market. The same argument applies for the product diversity as fluctuations in extensive margins  $N_t$  also impact expected utility through their mean and variance and through their covariance with demand shocks (third square bracket term). However, their impact depends crucially on the presence of love for product variety,  $\gamma$ .

We now discuss the role of the timing of entry, investment and production on the choice of exchange rate systems.

## 4 Contemporaneous production

Because firms repeatedly enter and exit markets, the number of product varieties - the extensive margin - varies across time. As a result, the choice of exchange rate systems is likely to depend on the changes in extensive margins. In this section we study a model where firms to enter at every time period and invest and produce in the same time period. In this case, although the economy may respond to shocks through changes in both intensive and extensive margins, shocks are absorbed only extensive margins when exchange rates are fixed.

The impact of monetary policies on output is determined as follows. As in Corsetti and Pesenti (2005, 2009), we define the *monetary stance* as  $\mu_t \equiv P_t C_t$ . The monetary stance is here derived from (4) as  $\mu_t = (M_t/\chi) (1 - E_t \Lambda_{t,t+1})$ , which, after substituting for  $\Lambda_{t,t+1}$ , yields the

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<sup>5</sup>For instance, one can make the following approximation for small demand shocks:  $E_0 \ln y_t \simeq \ln \bar{y} - \frac{1}{2} \text{var} \left( \frac{y_t}{\bar{y}} \right)$  and  $E_0 \ln N_t \simeq \ln \bar{N} - \frac{1}{2} \text{var} \left( \frac{N_t}{\bar{N}} \right)$  @where  $\bar{y}$  and  $\bar{N}$  denote the steady state value of intensive and extensive margins. @

following recursive identity:

$$\mu_t = \frac{M_t}{\chi} (1 - \beta \mu_t E_t \mu_{t+1}^{-1}).$$

This identity solves to

$$\mu_t = \frac{1}{\chi} \frac{1}{(1/M_t) + \sum_{s=1}^{\infty} \beta^s E_t (1/M_{t+s})}. \quad (20)$$

So, the current monetary stance is a function of the current and expected future money supply.

As a result, the exchange rate can be expressed in terms of the monetary stance as

$$\varepsilon_t = \frac{\alpha_t^* \mu_t}{\alpha_t \mu_t^*}.$$

In the domestic country, the equilibrium wages, extensive and intensive margins are then computed as (see more variables in Appendix A, Table A1)

$$W_t = \xi \left( E_{t-1} \mu_t^{1+\psi} \right)^{\frac{1}{1+\psi}}, \quad N_t = \frac{1}{\sigma f} \frac{\mu_t}{W_t} \quad \text{and} \quad y_t = \frac{\sigma - 1}{\sigma} \frac{\mu_t}{W_t N_t}$$

where

$$\xi \equiv \left( \kappa \frac{\theta}{\theta - 1} \right)^{\frac{1}{1+\psi}}$$

We can make several comments from those expressions. First, wages are sticky and depend on the expectation of the monetary stance. At given wages, extensive margins increase proportionally with the monetary stance. An expansion of domestic money supply stimulates current expenditure on consumption goods and increases local firms' profit, which triggers the entry of new product varieties. This effect is similar to the one discussed in Bergin and Corsetti (2008). Second, the expansion of the domestic money supply also stimulates production scales of incumbents but the latter are exactly cancelled out by the business stealing effect of new entrants. Indeed, given the above equalities, we get  $y_t = f(\sigma - 1)$ . Third, it can be shown that  $\left( E_{t-1} \mu_t^{1+\psi} \right)^{\frac{1}{1+\psi}}$  is an increasing function of  $\psi$  and the variance of  $\mu_t$ . Therefore, wages increase with weaker labor supply elasticity  $\psi^{-1}$  and larger variance in monetary stance when  $\psi > 0$ . A larger variance in  $\mu_t$  amplifies the fluctuations in consumers' product demands and therefore firms' labor demands. This entices workers to claim higher wages in compensation for future wage uncertainty, hence increases firms' costs. As a result of these higher wages, the number of firms falls. However, when the labor supply is infinitely elastic ( $\psi = 0$ ), the variability of monetary stance does not matter in wage setting behavior.

In a flexible exchange rate regime, the domestic and foreign money supply ( $M_t, M_t^*$ ) are constant for all time periods. The monetary stance is given as  $\mu_t = \mu_t^* = \mu_0$  where  $\mu_0$  is a constant. The exchange rate becomes  $\varepsilon_t = \alpha_t^* / \alpha_t$ . Replacing  $P_t C_t$  with  $\mu_t = \mu_0$  in the above

expressions, we can compute the equilibrium wage, extensive and intensive margins as

$$W_t = \mu_0 \xi, \quad N_t = \frac{1}{\sigma f \xi} \quad \text{and} \quad y_t = (\sigma - 1) f$$

In this regime, not only are wages but also extensive and intensive margins constant and independent of shock distributions. The exchange rate perfectly absorbs the effects of demand shocks on wages and margins. This is the allocation of production that would prevail in an economy without wage rigidities.

In a fixed exchange rate regime, the domestic and foreign money supply ( $M_t, M_t^*$ ) are set so that the exchange rate  $\varepsilon_t$  equals 1. This means that monetary authorities take procyclical monetary stances such as  $\mu_t = 2\mu_0\alpha_t$  and  $\mu_t^* = 2\mu_0\alpha_t^*$ . Replacing  $P_t C_t$  by  $\mu_t$  in the above expressions, we can compute wages, extensive and intensive margins as follows:

$$W_t = 2\mu_0 \xi A, \quad N_t = \frac{1}{\sigma f \xi} \frac{\alpha_t}{A} \quad \text{and} \quad y_t = (\sigma - 1) f$$

where

$$A \equiv \left( E_0 \alpha_t^{1+\psi} \right)^{\frac{1}{1+\psi}} \geq 1/2 \quad (21)$$

increases with stronger variance in  $\alpha_t$ . It is shown in Appendix B that  $A$  increases from  $1/2$  to  $\bar{\alpha}$  as  $\psi$  rises from zero to infinity. The wage is higher under fixed exchange rates because workers require a risk premium to compensate their higher work time volatility. We can make the following points from those expressions: First, wages are constant but depend on shock distributions (through  $A$ ). Wages coincide under flexible and fixed exchange rate regimes when labor supply is perfectly elastic ( $\psi = 0 \iff A = 1/2$ ). The wage is higher under fixed exchange rates because workers require a risk premium to compensate for labor demand fluctuations. Since  $W_t$  rises with  $A$ , wages increase with the shock variance and with the lower labor supply elasticity (larger  $\psi$ ). Second, only extensive margins respond to shocks because the business stealing effect of new entrants. Higher demand for local goods triggers firm entry under procyclical monetary policy. Finally, extensive margins fall with  $A$ . Indeed, a higher shock variance or a lower labor supply elasticity increases wages and therefore reduces firms' incentives to enter in the market.

Compared to the flexible exchange rate regime, the domestic expected welfare at  $t = 0$  under fixed exchange rates differs only from its extensive margins  $N_t$ . Hence, using (19), the condition under which a fixed exchange rate regime yields a higher welfare than a flexible one is given by

$$(1 + \gamma) [E_0 \alpha_t E_0 \ln(\alpha_t/A) + \text{cov}(\alpha_t, \ln \alpha_t/A)] > 0 \quad (22)$$



The first term in the bracketed term is negative represents the welfare loss under fixed exchange rates ( $E_0 \ln(\alpha_t/A) = E_0 \ln \alpha_t - E_0 \ln A < 0$ ). The second term is strictly positive and measures the gains from the congruence between domestic preferences and domestic product diversity. Because  $A$  is a constant, this term is equal to  $\text{cov}(\alpha_t, \ln \alpha_t)$ .

In expression (22), the choice of exchange rate regime is independent from the love for variety  $\gamma$ . Only the elasticity of labor supply elasticity  $\psi^{-1}$  matters through the terms  $A$ . Since  $A$  increases in  $\psi$ , the above expression falls below zero as  $\psi$  increases from zero to infinity. Therefore there exists a unique threshold  $\psi$  below which (22) is positive and above which it is negative.

**Proposition 1** *In the model with contemporaneous production, there exists a labor supply elasticity threshold  $\psi_0^{-1}$  such that a fixed exchange rate system is preferred for labor elasticities  $\psi^{-1}$  larger than  $\psi_0^{-1}$ .*

**Proof.** See Appendix B. ■

In this model with entry and contemporaneous production, intensive margins play no role because demand rises are fully absorbed by new entrants. As this may lack realism, we now discuss the model where extensive and intensive margins coexist because of production lags.

Figure 1 plots the welfare difference between fixed and flexible exchange rates as a function of the labor supply elasticity  $\psi^{-1}$ . The welfare difference under contemporaneous production is displayed with the solid blue curve. Flexible exchange rate regimes dominate when the value on the vertical axis exceeds one. The figure shows that they are preferred for high enough labor supply elasticities. Note that, as explained above, the choice of the regime is independent from the values of love for variety.

This result compares with the literature on currency union where entry is fixed. In particular, Devereux (2004) finds that fixed exchange rate systems are supported for high enough labor supply elasticities in a static model with perfect competition and two varieties (one produced in each country). Actually, the condition for an optimal currency union in the static model corresponds to our condition (22) in the absence of love for variety. Hence, the extensive margins in the present dynamic model play the same role of shock absorber as the intensive margins in the static model. Under CES preferences, the absence of love for variety implies that consumption fluctuations  $C_t$  are functions of national products  $N_t y_t$  and  $N_t^* y_t^*$  only (see (18)). It therefore does not matter whether shocks are absorbed by intensive or extensive margins.<sup>6</sup>

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<sup>6</sup>Finally, we note that our model does not have the same production and preference structure as Devereux'

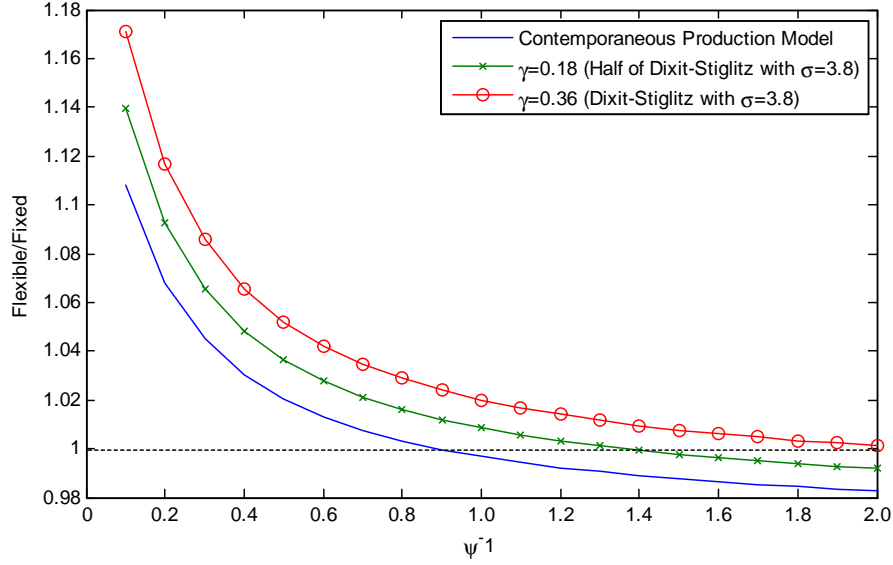


Figure 1: Welfare Advantage of Currency Area

## 5 Lagged production

Suppose now that firms enter and invest in the period before their production. As before, the exchange rate is a function of monetary stances,  $\varepsilon_t = (\alpha_t^*/\alpha_t)(\mu_t/\mu_t^*)$ , which allows us to compute the following equilibrium wages, extensive and intensive margins

$$W_t = \xi \phi \left( E_{t-1} \mu_t^{1+\psi} \right)^{\frac{1}{1+\psi}}, \quad N_{t+1} = \frac{\beta}{\sigma f} \frac{\mu_t}{W_t} \quad \text{and} \quad y_t = \frac{\sigma - 1}{\sigma} \frac{\mu_t}{W_t N_t},$$

where

$$\phi \equiv \left( \frac{\sigma - 1 + \beta}{\sigma} \right)^{\frac{\psi}{1+\psi}}$$

is a constant (see other variables in Appendix A, Table A2). The expansion of monetary stance boosts the current nominal expenditures and therefore the current incumbent firms' revenues and consumers' incentives to save. On the one hand, this expands the incumbent firms' current production scale,  $y_t$ . On the other hand, it also increases firms' discounted expected operational profits above the entry costs, which remains unchanged because of wage stickiness. As a result, more firms enter with the perspective to produce in the next period. Because entrants do not produce at the same time as incumbents, there is no business stealing effect following current

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because it includes fixed inputs in each period and a continuum of product varieties. To be comparable the static model should include those features. It can be shown that the condition for an optimal currency area remains the same as Devereux' only in the particular case where product diversity  $N_t$  and/or fixed input  $f$  tend to zero.

demand shocks.

In the flexible exchange rate regime, the domestic and foreign money supply are constant for all time periods so that the monetary stance is again  $\mu_t = \mu_t^* = \mu_0$  and the exchange rate is equal to  $\varepsilon_t = \alpha_t^*/\alpha_t$ . Replacing  $P_t C_t$  by  $\mu_t = \mu_0$  in the above expressions, we get

$$W_t = \mu_0 \xi \phi, \quad N_t = \frac{\beta \phi}{\sigma f \xi} \quad \text{and} \quad y_t = (\sigma - 1) \frac{f}{\beta}.$$

As in the contemporaneous production model, the exchange rate perfectly absorbs the effects of demand shocks on wages and margins so that margins remain constant.

In the fixed exchange rate regime, the domestic and foreign money supplies are set to maintain a fixed exchange rate  $\varepsilon_t = 1$ . Monetary stances are procyclical and then equal to  $\mu_t = 2\mu_0 \alpha_t$  and  $\mu_t^* = 2\mu_0 \alpha_t^*$ . Replacing  $P_t C_t$  by  $\mu_t$  in the above expressions, the wages, the equilibrium extensive and intensive margins are computed as follows:

$$W_t = 2\mu_0 \xi \phi A, \quad N_{t+1} = \frac{\beta}{\sigma f \xi} \frac{\alpha_t}{A} \quad \text{and} \quad y_t = (\sigma - 1) \frac{f}{\beta} \frac{\alpha_t}{\alpha_{t-1}}$$

As before, wages rise with the higher variance of the shocks (larger  $A$ ) and lower elasticity of labor supply (larger  $\psi$ ). Contrary to the contemporaneous production model, both the extensive and intensive margins here respond to shocks under fixed exchange rate regimes. The future extensive margins vary with current period shocks due to the procyclical feature of monetary policy. The intensive margins adapt to both the current and previous period demand shocks. They rise with the current shock,  $\alpha_t$ , because of the pro-active monetary policy and fall with the past demand shock,  $\alpha_{t-1}$ , because of the past monetary policy that boosted firms' entry.

We can now compare welfare under the two regimes. Welfare differences stem from both extensive and intensive margins. Hence, using (19), the fixed exchange rate regime supporting condition is given by

$$E_0 \alpha_t E_0 \ln \frac{\alpha_t}{\alpha_{t-1}} + \text{cov}(\alpha_t, \ln \frac{\alpha_t}{\alpha_{t-1}}) + (1 + \gamma) [E_0 \alpha_t E_0 \ln (\alpha_{t-1}/A) + \text{cov}(\alpha_t, \ln \alpha_{t-1})] > 0.$$

The first term reflects the impact of the mean and variance of intensive margins. Since  $E_0 \ln \alpha_t / (\alpha_{t-1}) = E_0 \ln \alpha_t - E_0 \ln \alpha_{t-1} = 0$  and since shocks are i.i.d, this term is nil. The second term measures the congruence of current preferences and supply of each domestic product. With i.i.d. shocks, this term simplifies to  $\text{cov}(\alpha_t, \ln \alpha_t) > 0$ , which reflects the benefit of a congruence between preferences and product supplies under fixed exchange rate regimes. The last square bracket expresses the same trade-off as in the case of contemporaneous production. The first term is the mean and variance effect of extensive margins. This is a loss that is weakened by a higher labor supply elasticity (larger  $\psi^{-1}$ ) through a lower  $A$ . Since demand shocks are

i.i.d., the second term in the bracket is nil. Past movements in firm entry and exit, hence the resulting fluctuations in product diversity, cannot be related to present preferences and do not bring any congruence benefit.

The above condition simplifies to

$$(1 + \gamma) [E_0 \alpha_t E_0 \ln (\alpha_{t-1}/A)] + \text{cov}(\alpha_t, \ln \alpha_t) > 0 \quad (23)$$

which decreases in  $A$  and therefore in  $\psi$ . By the same argument as for Proposition 1, the expression (23) accepts a unique root  $\psi_1$ . Since expression (23) is smaller than (22) by the term  $\gamma \text{cov}(\alpha_t, \ln \alpha_t/A) > 0$ , which increases in  $\gamma$ , this root  $\psi_1$  is smaller than  $\psi_0$  and that increases with larger  $\sigma$ . The roots are identical in the absence of love for variety ( $\gamma = 0$ ).

**Proposition 2** *In a model with lagged production, there exists a labor supply elasticity threshold  $\psi_1^{-1}$  such that a fixed exchange rate system is preferred for  $\psi^{-1} > \psi_1^{-1}$ . The threshold  $\psi_1^{-1}$  is larger than  $\psi_0^{-1}$  and falls as love for product diversity  $\gamma$  rises.*

To illustrate the proposition, the colored curves in Figure 1 display the welfare difference between the exchange rate regimes with lagged production. The red and green curve shows this difference respectively when the parameter for love for variety takes the Dixit-Stiglitz' value,  $\gamma = 1/(\sigma - 1)$  and when it takes the half of this value. The second value roughly approximates Ardelean's (2006) estimate of  $\gamma$  about 42% of  $1/(\sigma - 1)$ . As stated above, the black curve here corresponds to the case where  $\gamma = 0$ . We set the elasticity of product substitution  $\sigma$  to 3.8 according to Bernard et al. (2003), which gives the parameters  $\gamma \in \{0, 0.18, 0.36\}$ . At a given labor supply elasticity, fixed exchange rate regime becomes harder to support when the love for product variety rises.

Accordingly, a fixed exchange rate system is less likely to be supported when production lags behind entry and investment. The reason is however not trivial because production lags change the nature of the costs and benefits of exchange rate systems. Indeed, in this lagged production model, a fixed exchange rate system can improve welfare because the better congruence between preferences and product supplies outweighs the cost of higher variance in extensive margins. By contrast, in the contemporaneous production model, a fixed exchange rate system improves welfare when the better congruence between preferences and product diversity outweigh the cost of higher volatility in product diversity.

## 6 Conclusion

This paper studies the role of intensive and extensive margins on the choice of fixed or flexible exchange rate regimes. The literature has focused on how exchange rates and intensive margin absorb macro-economic shocks and mitigate production and employment volatility. The present paper discusses the relevance of extensive margins as shock absorbers. In a first contemporaneous production model where entry and investment take place in the same time period, we show that intensive margins do not fluctuate and that only extensive margins vary following a demand shock in fixed exchange rate regimes. The choice for a fixed exchange rate regime then results from the balance between the cost of a lower level and a higher variance of product diversity and the benefit of the better congruence between preferences and product diversity. Fixed exchange rate regimes are preferred for high enough labor supply elasticities because more flexible labor supplies represents better shock absorbers. We then discuss the same model where production lags behind entry. In this case, both intensive and extensive margins may fluctuate. More particularly, extensive margins under fixed exchange rates have a negative contribution to welfare because they lead to low level and a high volatility of product diversity. However, intensive margins bring a positive welfare contribution because they align consumption with preferences. Fixed exchange rate regimes are then less likely to be supported when production lags exist. They also perform worse when consumers express a higher preference for product diversity.

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## Appendix A: Optimal choice of households

The problem can be stated in terms of the following optimization of the Lagrangian function  $\mathcal{L}_0(i)$ :

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ U_t + \lambda_t(i) [ w_t(i) l_t(i) + \int_{\omega \in \mathcal{X}_t} s_{t-m}(i, \omega) d_t(\omega) d\omega + M_{t-1}(i) - P_t C_t(i) - \int_{\omega \in \mathcal{X}_{t+m}} s_t(i, \omega) q_t(\omega) d\omega - M_t ] \}$$

with respect to  $\{x_t(i, \omega), z_t(j, \nu), M_t(i), s_t(i, \omega), w_t(i)\}_{t=0}^{\infty}$  where  $\lambda_t(i)$  denotes the Lagrangian multiplier associated with the flow budget constraint at time  $t$ . Note that  $P_t C_t(i) = \int_{\omega \in \mathcal{X}_t} p_t(\omega) x_t(i, \omega) d\omega + \int_{v \in \mathcal{Z}_t} p_t(v) z_t(i, v) dv$ .

The first order condition with respect to  $C_t(i)$  yields

$$\frac{1}{C_t(i)} - \lambda_t(i)P_t = 0 \quad (24)$$

So,  $\lambda_t(i)$  represents the marginal utility stemming from one additional unit of nominal wealth.

The above expression is identical for both models with  $m = 0, 1$ .

The first order condition with respect to  $M_t(i)$  yields

$$\frac{\chi}{M_t(i)} - \lambda_t(i) + \beta E_t \lambda_{t+1}(i) = 0 \quad (25)$$

The first order condition with respect to  $s_t(i, \omega)$  gives

$$-\lambda_t(i)q_t(\omega) + \beta E_t d_{t+1}(\omega) \lambda_{t+1}(i) = 0$$

The marginal utility of nominal wealth at  $t$  is equal to the discounted marginal utility at  $t + 1$ .

This condition is redundant when  $m = 0$ .

The household also sets the future wage  $w_{t+1}(i)$  at  $t$  knowing the demand function for her labor service  $l_{t+1}(i)$ . The first order condition with respect to  $w_{t+1}(i)$  yields

$$\kappa \theta \frac{E_t l_{t+1}(i)^{1+\psi}}{w_{t+1}(i)} - (\theta - 1) E_t [\lambda_{t+1}(i) l_{t+1}(i)] = 0$$

Accordingly, the expected disutility of a marginal work effort is equal to the expected consumption utility of the associated marginal wage increase.

We can summarize the solutions for the contemporaneous and lagged production models  $m = 0, 1$  for any exchange rate in the following tables:

Home variables	Foreign variables
$C_t = \alpha_t \Phi_t$	$C_t^* = (1 - \alpha_t) \Phi_t$
$\Phi_t \equiv N_t^{(1+\gamma)\alpha_t} N_t^{*(1+\gamma)(1-\alpha_t)} \left(\frac{y_t}{\alpha_t}\right)^{\alpha_t} \left(\frac{y_t^*}{1-\alpha_t}\right)^{1-\alpha_t}$	$\varepsilon_t = \frac{1-\alpha_t}{\alpha_t} \frac{\mu_t}{\mu_t^*}$
$l_t = \frac{\mu_t}{W_t}$	$l_t^* = \frac{\mu_t^*}{W_t^*}$
$N_t = \frac{1}{\sigma} \frac{\mu_t}{W_t f}$	$N_t^* = \frac{1}{\sigma} \frac{\mu_t^*}{W_t^* f^*}$
$y_t = \frac{\sigma-1}{\sigma} \frac{\mu_t}{W_t N_t} = (\sigma - 1) f$	$y_t^* = \frac{\sigma-1}{\sigma} \frac{\mu_t^*}{W_t^* N_t^*} = (\sigma - 1) f^*$
$p_t = \frac{\sigma}{\sigma-1} W_t$	$p_t^* = \frac{\sigma}{\sigma-1} W_t^*$
$d_t = \frac{1}{\sigma} \frac{\mu_t}{N_t}$	$d_t^* = \frac{1}{\sigma} \frac{\mu_t^*}{N_t^*}$
$q_t = W_t f$	$q_t^* = W_t^* f^*$
$W_t = \xi \left(E_{t-1} \mu_t^{1+\psi}\right)^{\frac{1}{1+\psi}}$	$W_t^* = \xi \left(E_{t-1} \mu_t^{*1+\psi}\right)^{\frac{1}{1+\psi}}$
$\mu_t = P_t C_t$	$\mu_t^* = P_t^* C_t^*$



Table A1: Solution in contemporaneous entry model ( $m = 0$ )

Home variables	Foreign variables
$C_t = \alpha_t \Phi_t$	$C_t^* = (1 - \alpha_t) \Phi_t$
$\Phi_t \equiv N_t^{(1+\gamma)\alpha_t} N_t^{*(1+\gamma)(1-\alpha_t)} \left(\frac{y_t}{\alpha_t}\right)^{\alpha_t} \left(\frac{y_t^*}{1-\alpha_t}\right)^{1-\alpha_t}$	$\varepsilon_t = \frac{1-\alpha_t}{\alpha_t} \frac{\mu_t}{\mu_t^*}$
$l_t = \left(\frac{\sigma-1+\beta}{\sigma}\right) \frac{\mu_t}{W_t}$	$l_t^* = \left(\frac{\sigma-1+\beta}{\sigma}\right) \frac{\mu_t^*}{W_t^*}$
$N_t = \frac{\beta}{\sigma} \frac{\mu_{t-1}}{W_{t-1} f}$	$N_t^* = \frac{\beta}{\sigma} \frac{\mu_{t-1}^*}{W_{t-1}^* f^*}$
$y_t = \frac{\sigma-1}{\sigma} \frac{\mu_t}{W_t N_t}$	$y_t^* = \frac{\sigma-1}{\sigma} \frac{\mu_t^*}{W_t^* N_t^*}$
$p_t = \frac{\sigma}{\sigma-1} W_t$	$p_t^* = \frac{\sigma}{\sigma-1} W_t^*$
$d_t = \frac{1}{\sigma} \frac{\mu_t}{N_t}$	$d_t^* = \frac{1}{\sigma} \frac{\mu_t^*}{N_t^*}$
$q_t = W_t f$	$q_t^* = W_t^* f^*$
$W_t = \xi \phi \left(E_{t-1} \mu_t^{1+\psi}\right)^{\frac{1}{1+\psi}}$	$W_t^* = \xi \phi \left(E_{t-1} \mu_t^{*1+\psi}\right)^{\frac{1}{1+\psi}}$
$\mu_t = P_t C_t$	$\mu_t^* = P_t^* C_t^*$

Table A2: Solution in lagged production model ( $m = 1$ )

## Appendix B: Proof of Proposition 1

(i) We need to show that  $A$  is an increasing function of  $\psi$  and is negative for large  $\psi$ . Let  $G : [1 - \bar{\alpha}, \bar{\alpha}] \rightarrow [0, 1]$  be the cumulative distribution of  $\alpha_t$  where  $\bar{\alpha} \in [1/2, 1]$  is the upper bound of the distribution. We first show that  $dA/d\psi > 0$ . Indeed, let  $f(\psi) \equiv A^{1+\psi} = \int \alpha^{1+\psi} dG(\alpha)$ , which is lower than one because  $\alpha_t < 1$  for all  $\alpha_t$  and is an increasing function as  $f'(\psi) = (1 + \psi) \int \alpha^\psi dG(\alpha) > 0$ . Then, we compute that  $d \ln A / d\psi = (d/d\psi) [\ln f(\psi)^{1/(1+\psi)}] = (d/d\psi) [(1 + \psi)^{-1} \ln f(\psi)] = -(1 + \psi)^{-2} \ln f(\psi) + (1 + \psi)^{-1} f'(\psi) / f(\psi)$ , which is positive because each term is positive in the last expression. Since  $d \ln A / d\psi = A^{-1} dA / d\psi$ , it must be that  $dA / d\psi > 0$ . Second, expanding the covariance term, the bracket in expression (22) is equal to  $E_0 \alpha_t E_0 \ln \alpha_t - E_0 \alpha_t E_0 \ln A + E_0 \alpha_t \ln \alpha_t - E_0 \alpha_t E_0 \ln \alpha_t$ , which simplifies to  $E_0 \alpha_t (\ln \alpha_t - \ln A)$ . The latter expression is negative because  $\ln \alpha_t < \ln A = \ln \bar{\alpha}$  for  $\psi \rightarrow \infty$ . Indeed, we successively get  $\lim_{\psi \rightarrow \infty} \ln A = \lim_{\psi \rightarrow \infty} \ln [E_0 \alpha^{1+\psi}]^{\frac{1}{1+\psi}} = \lim_{\psi \rightarrow \infty} \ln \left[ \int_{1-\bar{\alpha}}^{\bar{\alpha}} \alpha^{1+\psi} dG(\alpha) \right]^{\frac{1}{1+\psi}} = \lim_{\psi \rightarrow \infty} \ln \left[ \int_{1-\bar{\alpha}}^{\bar{\alpha}} (\alpha / \bar{\alpha})^{1+\psi} \bar{\alpha}^{1+\psi} dG(\alpha_t) \right]^{\frac{1}{1+\psi}}$ . Since  $\lim_{\psi \rightarrow \infty} (\alpha_t / \bar{\alpha}_t)^{1+\psi} = 0$  for any  $\alpha_t < \bar{\alpha}_t$ , the latter expression becomes  $\lim_{\psi \rightarrow \infty} \ln [\bar{\alpha}^{1+\psi} g(\bar{\alpha})]^{\frac{1}{1+\psi}} = \ln \bar{\alpha} + \lim_{\psi \rightarrow \infty} [1 / (1 + \psi)] \ln g(\bar{\alpha}) = \ln \bar{\alpha}$ .

(ii) We need to show that  $E_0 \alpha_t \ln (\alpha_t / E_0 \alpha_t) > 0$ . Indeed, this equivalent to  $E_0 2\alpha_t \ln 2\alpha_t > 0$ . Since  $x \geq \ln(1+x) \geq x - x^2/2$ , we get  $2\alpha_t (2\alpha_t - 1) \geq 2\alpha_t \ln 2\alpha_t \geq 2\alpha_t (2\alpha_t - 1) - 2\alpha_t (2\alpha_t - 1)^2 / 2$ . The LHS of those conditions can be written as  $(2\alpha_t - 1)^2 + (2\alpha_t - 1)$ ,

which expected value is  $E_0(2\alpha_t - 1)^2 = 4\text{var}(\alpha_t)$ . The RHS can be written as  $(2\alpha_t - 1)^2 + (2\alpha_t - 1) - (2\alpha_t - 1)^3/2 - (2\alpha_t - 1)^2/2$ , which expected value is  $E_0(2\alpha_t - 1)^2/2 = 2\text{var}(\alpha_t)$  because  $E_0(2\alpha_t - 1)^3 = 0$  for any symmetric distribution of shock  $\alpha_t$ . Then,  $4\text{var}(\alpha_t) \geq E_0 2\alpha_t \ln 2\alpha_t \geq 2\text{var}(\alpha_t)$ , so that  $E_0 2\alpha_t \ln 2\alpha_t > 0$ .